Published by Institute of Physics Publishing for SISSA

Received: *June 6, 2006*
Accepted: *October 3, 2006*
Published: *October 24, 2006*

Schnabl's $\mathcal{L}_0$ operator in the continuous basis

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Abstract: Following Schnabl's analytic solution to string field theory, we calculate the operators $\mathcal{L}_0, \mathcal{L}_0^\dagger$ for a scalar field in the continuous κ basis. We find an explicit and simple expression for them that further simplifies for their sum, which is block diagonal in this basis. We generalize this result for the bosonized ghost sector, verify their commutation relation and relate our expressions to wedge state representations.

Keywords: String Field Theory, Bosonic Strings.

Contents

1. Introduction and summary

Recently Schnabl found an analytic non trivial solution [1] to open bosonic cubic string field theory [2]. This solution is universal [3], and therefore it was written in a universal way, independent of the CFT of the matter sector. The only requirement from the matter sector is that it should have central charge 26 to cancel the central charge of the b, c ghost system.

Yet, many applications and generalizations of the solution do depend on the matter sector. Foremost, the proof of Sen's first conjecture, according to which the height of the potential equals the tension of a D25-brane, obviously assumes that the matter sector consists of 26 scalars with Neumann boundary conditions. Yet, for this calculation, the dependence on the matter sector simply amounts to integrating over the zero-modes, which gives the needed 26 dimensional volume factor. Therefore, Schnabl was able to prove Sen's first conjecture within the universal basis (up to some subtleties that were clarified in [4, 5]).

Other generalizations, like finding lump solutions [6 − 8] or studying the close string spectrum around the solution, should also depend on the matter sector. Schnabl's solution may also have relevance to other, background dependent constructions, such as the evaluation of (off-shell) string amplitudes [9, 10]. All this calls for a study of the scalar field in Schnabl's formulation.

To solve the equations of motion of string field theory, Schnabl made a specific choice of gauge and coordinates. The zeroth order Virasoro generator in these coordinates is

$$\mathcal{L}_0 = \tan \circ L_0 = \oint \frac{d\tilde{z}}{2\pi i} \tilde{z} T_{\tilde{z}\tilde{z}}(\tilde{z}) = L_0 - \sum_{n=1}^{\infty} \frac{2(-1)^n}{4n^2-1} L_{2n} \,. \tag{1.1}$$

This operator and its conjugate play a prominent role in the solution. The simplest way to write this operator for a scalar field would be to use oscillators in the same coordinate system,

$$\mathcal{L}_0 = \frac{\tilde{\alpha}_0^2}{2} + \sum_{n=1}^{\infty} \tilde{\alpha}_{-n} \tilde{\alpha}_n \,, \qquad \tilde{\alpha}_n = \tan \circ \alpha_n = \oint \frac{d\tilde{z}}{2\pi i} \tilde{z}^n \partial X(\tilde{z}) \,. \tag{1.2}$$

The downside of this approach is that the BPZ conjugate operations become complicated. For example, the relation between $\tilde{\alpha}_n^\dagger$ and $\tilde{\alpha}_n$ is no longer simple. Also, the vacuum state annihilated to the right by all $\tilde{\alpha}_{n>0}$ remains the same as can be seen from the relation $U_{\tan} |0\rangle = |0\rangle$. The conjugate state, the one annihilated to the left by all $\tilde{\alpha}_{n<0}$, on the other hand, is the sliver state $\langle 0| U_{\tan}^{-1} = \langle S|$. This can also be seen from the generalization of the squeezed state expression of [11] for the new operators,

$$\langle f| = \langle 0| \exp\left(\frac{1}{2} \tilde{\alpha}_n s_{nm} \tilde{\alpha}_m\right) , \qquad s_{nm} = \frac{1}{nm} \oint \frac{dz}{2\pi i} \frac{dw}{2\pi i} z^{-n} w^{-m} \frac{f'(z) f'(w)}{\sin(f(z) - f(w))^2} \,. \tag{1.3}$$

Here the trivial map $f(z) = z$ gives the sliver.

Another way to avoid using an explicit expression for $\mathcal{L}_0$ is to switch the solution to the Siegel gauge. Since $L_0 = U_{\tan}^{-1} \mathcal{L}_0 U_{\tan}$, one could naively define,

$$L_0^\ddagger \equiv U_{\tan}^{-1} \mathcal{L}_0^\dagger U_{\tan} \,. \tag{1.4}$$

Then, this operator will satisfy the desired algebra $[L_0, L_0^\ddagger] = L_0 + L_0^\ddagger$ by construction and one could use these operators to build a solution equivalent to Schnabl's solution in the Siegel gauge. However, trying to calculate $L_0^\ddagger$ leads to diverging results, and we did not find a way to make sense out of this operator.

An alternative route for simplifying Schnabl's operators stems from their relations to wedge states [12 − 15]. That Schnabl's solution is related to wedge states is clear, both because they are explicitly used in the construction of the solution and because his gauge choice is implemented via a conformal transformation which is the inverse of the sliver transformation. Wedge states are especially easy to deal with in the continuous basis [16], as they are squeezed states whose defining matrix is diagonal in this basis. In fact, they are the only surface states with this property [17, 18].

The wedge states and the conformal transformation are both generated by the operators $\mathcal{L}_0, \mathcal{L}_0^\dagger$. Thus, one would expect that these operators would have a nice form in the continuous basis. To be more concrete, we recall that the wedge state $|n\rangle$ can be represented as,

$$|n\rangle = e^{\log\left(\frac{2}{n}\right) \mathcal{L}_0^\dagger} |0\rangle \,, \tag{1.5}$$

and also as,

$$|n\rangle = e^{\left(-\frac{n-2}{2}(\mathcal{L}_0+\mathcal{L}_0^\dagger)\right)}\,|0\rangle\,. \tag{1.6}$$

On the other hand, this should also be equal to [13, 16],

$$|n\rangle = e^{\int_0^\infty d\kappa\, T_n(\kappa) a_\kappa^\dagger a_{-\kappa}^\dagger}\,|0\rangle\,, \tag{1.7}$$

where

$$T_n(\kappa) = \frac{e^{\frac{\kappa\pi}{2}(n-1)} - e^{\frac{\kappa\pi}{2}}}{1 - e^{\frac{\kappa\pi}{2}n}} = -\frac{\sinh\left(\frac{\kappa\pi}{4}(n-2)\right)}{\sinh\left(\frac{\kappa\pi n}{4}\right)}\,. \tag{1.8}$$

Thus, we expect a simple representation for these operators in the continuous basis.

Yet another hint for the natural description of these operators in the continuous basis comes from the commutation relation

$$[\mathcal{L}_0 + \mathcal{L}_0^\dagger, K_1] = 0\,. \tag{1.9}$$

This relation implies that the bi-linear term of $\mathcal{L}_0 + \mathcal{L}_0^\dagger$ is diagonal and suggests that the quadratic terms are simple.

Indeed we find,

$$\mathcal{L}_0 + \mathcal{L}_0^\dagger = \frac{\pi}{2}\int_{-\infty}^{\infty}\frac{d\kappa}{\mathcal{N}(\kappa)}\left(2\cosh\left(\frac{\kappa\pi}{2}\right)a_\kappa^\dagger a_\kappa + a_\kappa^\dagger a_{-\kappa}^\dagger + a_\kappa a_{-\kappa}\right). \tag{1.10}$$

Actually, it is very reassuring that we get such a simple and well behaved result considering the singular behaviour of the Virasoro operators in the continuous basis [19 − 21]. It seems that all the singularities conspire to cancel for this specific combination. The $\mathcal{L}_0$ operator itself is slightly less simple in the sense that in addition to the δ-function contributions it also has δ' contributions,

$$\mathcal{L}_0 = \int_{-\infty}^{\infty} d\kappa d\kappa' \left(\left(\frac{\kappa\pi}{4}\coth\left(\frac{\kappa\pi}{2}\right)\delta(\kappa-\kappa') + \frac{\kappa+\kappa'}{2}\delta'(\kappa-\kappa')\right)a_\kappa^\dagger a_{\kappa'} + \frac{\pi\delta(\kappa+\kappa')}{2\mathcal{N}(\kappa)}a_\kappa a_{\kappa'}\right). \tag{1.11}$$

These results are derived in section 2. We also derive in this section the non-zero momentum sector and bosonized ghost form of $\mathcal{L}_0, \mathcal{L}_0^\dagger$. In section 3 we verify that our expressions indeed satisfy the commutation relation,

$$[\mathcal{L}_0, \mathcal{L}_0^\dagger] = \mathcal{L}_0 + \mathcal{L}_0^\dagger\,. \tag{1.12}$$

We find that the commutation relation holds, up to regularization subtleties. When the central charge is not zero there is an additional infinite constant on the r.h.s of (1.12) [15]. These infinities cancel between the matter and the ghost sector, but in the oscillator regularization scheme we are left with a residual finite constant. This is reminiscent of the description of wedge states and string vertices in the continuous basis [22, 20, 23 − 25]. Section 4 is devoted to relating the different wedge state representations (1.5), (1.6), (1.7). We conclude and suggest future directions in section 5.

JHEP10(2006)067

2. The $\mathcal{L}_0, \mathcal{L}_0^\dagger$ operators in the continuous basis

The continuous basis is the basis that diagonalizes K_1 [16]. For a scalar field the creation and annihilation operators transform as

$$a_n^\dagger = \int_{-\infty}^{\infty} d\kappa \frac{v_n^\kappa}{\sqrt{\mathcal{N}(\kappa)}} a_\kappa^\dagger \,, \qquad a_n = \int_{-\infty}^{\infty} d\kappa \frac{v_n^\kappa}{\sqrt{\mathcal{N}(\kappa)}} a_\kappa \,. \tag{2.1}$$

The transformation matrix v_n^κ is defined by the generating function,

$$f_\kappa(z) = \frac{1 - e^{-\kappa \tan^{-1} z}}{\kappa} \equiv \sum_{n=1}^{\infty} \frac{v_n^\kappa}{\sqrt{n}} z^n \,, \tag{2.2}$$

and the normalization factor turns out to be [26]

$$\mathcal{N}(\kappa) = \frac{2}{\kappa} \sinh\left(\frac{\kappa\pi}{2}\right) . \tag{2.3}$$

The new creation and annihilation operators obey the commutation relation

$$[a_\kappa, a_{\kappa'}^\dagger] = \rho(\kappa, \kappa') \,, \tag{2.4}$$

where $\rho(\kappa, \kappa') = \delta(\kappa - \kappa')$ is the spectral density. For $\kappa = \kappa'$ there are also finite contributions which can be relevant. We therefore use level truncation to regularize the $\delta(0)$ contribution,

$$\rho(\kappa) \equiv \rho(\kappa, \kappa) = \lim_{\ell \to \infty} \frac{1}{2\pi} \sum_{n=1}^{\ell/2} \frac{1}{n} + \frac{4\log(2) - 2\gamma - \Psi(\frac{i\kappa}{2}) - \Psi(-\frac{i\kappa}{2})}{4\pi} \,. \tag{2.5}$$

2.1 A direct evaluation of $\mathcal{L}_0, \mathcal{L}_0^\dagger$

Virasoro generators cannot be represented in the continuous basis by functions, or even by usual delta functions [19]. This difficulty was addressed in [20, 21], where it was shown that the Virasoro generators can be represented by more general distributions, i.e. delta functions with complex arguments [20, 27]. The positive Virasoro modes are given by,

$$L_m = \frac{1}{2} \int_{-\infty}^{\infty} \frac{d\kappa\, d\kappa' \, a_\kappa a_{\kappa'}}{\sqrt{\mathcal{N}(\kappa)\mathcal{N}(\kappa')}} h_m^{\kappa,\kappa'} + \int_{-\infty}^{\infty} \frac{d\kappa d\kappa' a_\kappa^\dagger a_{\kappa'}}{\sqrt{\mathcal{N}(\kappa)\mathcal{N}(\kappa')}} g_m^{\kappa,\kappa'} \,, \tag{2.6}$$

where we refer to the two terms as quadratic and bi-linear, respectively. The coefficients of these terms, $g_m^{\kappa,\kappa'}, h_m^{\kappa,\kappa'}$ are,

$$\begin{aligned} g_m^{\kappa,\kappa'} =& \sinh\left(\frac{\kappa'\pi}{2}\right)\Big(\frac{q_m(\kappa_-)}{\sinh\left(\frac{\kappa_- \pi}{2}\right)} + \\ & \frac{i^m \delta(\kappa_- - 2i) - (-i)^m \delta(\kappa_- + 2i)}{2i} - m \sin\big(\frac{m\pi}{2}\big)\delta(\kappa_-)\Big), & (2.7) \\ h_m^{\kappa,\kappa'} =& q_m(\kappa_+) \,, & (2.8) \end{aligned}$$

where,

$$\kappa_\pm \equiv \kappa' \pm \kappa\,, \tag{2.9}$$

and the coefficients depend on,

$$q_m(\kappa) = \frac{1}{2\pi i}\oint \frac{e^{-\kappa \tan^{-1}(z)}dz}{z^{m-1}(1+z^2)^2}\,. \tag{2.10}$$

The negative Virasoro generators are simply given by conjugation.

We can now calculate $\mathcal{L}_0$ (1.1). First, we note that all the delta function contributions cancel between the L_0 part and the sum. Thus, we are only left with the evaluation of,

$$Q(\kappa) \equiv -2\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}q_{2n}(\kappa)\,. \tag{2.11}$$

This would give both the quadratic and the bi-linear coefficient, since they are both simple functions of $Q(\kappa_\pm)$.

We first observe that $Q(\kappa)$ is singular for $\kappa = 0$ since, $(-1)^n q_{2n}(0) = -n$, and so the sum behaves asymptotically as the harmonic sum. On the other hand, for $\kappa \neq 0$, an oscillatory behaviour is superposed on the linear divergence of the coefficients and so we would expect that the series would converge. The above suggests a component in $Q(\kappa)$ which is proportional to $\delta(\kappa)$. This would also fit our intuition, according to which $\mathcal{L}_0, \mathcal{L}_0^\dagger$ should have a simple form in the continuous basis.

We first use the symmetry of (2.10) for even integers to replace the exponent by a cosh and then integrate twice by parts to get,

$$q_{2n} = \frac{2n(2n-1)}{2\pi i}\oint \frac{\kappa\cosh\left(\kappa\tan^{-1}(z)\right)+2z\sinh\left(\kappa\tan^{-1}(z)\right)}{\kappa\left(\kappa^2+4\right)z^{2n+1}}dz. \tag{2.12}$$

In order to perform the sum we have to change the integration contour. We use the results for these integrals from [28, 20], and again integrate one of the summands by parts to get,

$$q_{2n} = \frac{2(-1)^{n+1}n(4n^2-1)\sinh\left(\frac{\kappa\pi}{2}\right)}{\kappa(\kappa^2+4)\pi}\int_{-\infty}^{\infty}\frac{\cos(\kappa u)}{\cosh^4(u)}\tanh^{2n-2}(u)du\,. \tag{2.13}$$

Plugging this result into (2.11) we get,

$$Q(\kappa) = \frac{4\sinh\left(\frac{\kappa\pi}{2}\right)}{\pi\kappa\left(\kappa^2+4\right)}\sum_{n=1}^{\infty}n\int_{-\infty}^{\infty}\frac{\cos(\kappa u)}{\cosh^4(u)}\tanh^{2n-2}(u)du\,. \tag{2.14}$$

We can now interchange the order of summation and integration and use

$$\sum_{n=1}^{\infty}n\tanh^{2n-2}(u) = \cosh^4(u)\,, \tag{2.15}$$

to get

$$Q(\kappa) = \frac{4\sinh\left(\frac{\kappa\pi}{2}\right)}{\pi\kappa\left(\kappa^2+4\right)}\int_{-\infty}^{\infty}\cos(\kappa u)du = \frac{4\sinh\left(\frac{\kappa\pi}{2}\right)}{\pi\kappa\left(\kappa^2+4\right)}2\pi\delta(\kappa) = \pi\delta(\kappa)\,. \tag{2.16}$$

JHEP10(2006)067

Substituting this result into the expression for h (2.8) we get that the term in $\mathcal{L}_0$ which is quadratic in annihilation operators is,

$$\mathcal{L}_{0,\text{quad}} = \pi \int_0^\infty \frac{a_\kappa a_{-\kappa}}{\mathcal{N}(\kappa)} d\kappa \,. \tag{2.17}$$

The term bilinear in creation and annihilation operators is more problematic. Here we formally find,

$$\mathcal{L}_{0,\text{bi-lin}} = 2 \int_{-\infty}^\infty \frac{\sinh\left(\frac{\kappa'\pi}{2}\right)}{\sqrt{\mathcal{N}(\kappa)\mathcal{N}(\kappa')}} \frac{\delta(\kappa-\kappa')}{\kappa'-\kappa} a_\kappa^\dagger a_{\kappa'} d\kappa d\kappa' \,. \tag{2.18}$$

This expression contains a very singular distribution that does not make sense unless it multiplies an expression with a zero at $\kappa = \kappa'$. We can, however, define it by a principal part prescription. We would later justify this choice. Integration by part and some basic manipulations give,

$$\mathcal{L}_{0,\text{bi-lin}} = \int_{-\infty}^\infty \left(\frac{\kappa\pi}{4} \coth\left(\frac{\kappa\pi}{2}\right) \delta(\kappa-\kappa') + \frac{\kappa+\kappa'}{2} \delta'(\kappa-\kappa')\right) a_\kappa^\dagger a_{\kappa'} d\kappa d\kappa' \,. \tag{2.19}$$

We see that the form of $\mathcal{L}_{0,\text{bi-lin}}$ almost matches our naive expectation, except for the appearance of a $\delta'(\kappa-\kappa')$ term.

Finally, we want to write the sum, $\mathcal{L}_0 + \mathcal{L}_0^\dagger$ explicitly. Its quadratic parts are just those of $\mathcal{L}_0^\dagger$ ($\mathcal{L}_0$) for the creation (annihilation) operators (2.17). For the bi-linear part, we have to sum two expressions. Here, the antisymmetric part cancels and so,

$$(\mathcal{L}_0 + \mathcal{L}_0^\dagger)_{\text{bi-lin}} = \int_{-\infty}^\infty d\kappa \frac{\kappa\pi}{2} \coth\left(\frac{\kappa\pi}{2}\right) a_\kappa^\dagger a_\kappa \,, \tag{2.20}$$

which is diagonal as it should be.

2.2 An alternative (half-string) evaluation of $\mathcal{L}_0 + \mathcal{L}_0^\dagger$

A useful representation of $\mathcal{L}_0 + \mathcal{L}_0^\dagger$ is given in eq. (2.44,2.45) of [1],

$$\mathcal{L}_0 + \mathcal{L}_0^\dagger = \frac{2}{\pi}(K_1 - 2K_1^R) = \frac{2}{\pi}(-K_1 - 2K_1^L) \,, \tag{2.21}$$

where,

$$K_1 = L_1 + L_{-1} \,, \tag{2.22}$$

is the operator defining the continuous basis and $K_1^{L,R}$ are its left and right parts in the half-string formulation. The operator K_1 is trivially diagonal in the κ basis and the transformation to half-string basis amounts to mixing $\pm\kappa$ [27, 29]. These facts imply the block diagonal form of $\mathcal{L}_0 + \mathcal{L}_0^\dagger$. We now want to evaluate it directly from (2.21).

The operator K_1 is given in the κ basis by,

$$K_1 = -\int_{-\infty}^\infty d\kappa \kappa a_\kappa^\dagger a_\kappa = \int_0^\infty d\kappa \kappa \left(a_{-\kappa}^\dagger a_{-\kappa} - a_\kappa^\dagger a_\kappa\right) . \tag{2.23}$$

We follow here the conventions of [29] and define the transformation to the continuous half-string basis by the Bogoliubov transformation,

$$\begin{pmatrix} a^l_\kappa \\ a^r_\kappa \end{pmatrix} = W \begin{pmatrix} a_{-\kappa} \\ a_\kappa \end{pmatrix} + U \begin{pmatrix} a^\dagger_{-\kappa} \\ a^\dagger_\kappa \end{pmatrix} . \tag{2.24}$$

Here W, U can be any pair of matrices built from block diagonal rank-one projectors as explained in [29]. For the sliver,

$$W = \frac{1}{\sqrt{1-e^{-\kappa\pi}}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad U = \frac{e^{-\frac{\kappa\pi}{2}}}{\sqrt{1-e^{-\kappa\pi}}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \tag{2.25}$$

Substituting the inverse transformation ($W \to W$, $U \to -U$) in the definition of K_1, we get its form in the (sliver) half-string basis,

$$K^h_1 = \int_0^\infty d\kappa\kappa \left(a^{l\,\dagger}_\kappa a^l_\kappa - a^{r\,\dagger}_{-\kappa} a^r_{-\kappa} \right) . \tag{2.26}$$

The simple κ dependence is unique to the sliver basis. Other projectors would result in more complicated expressions that would also contain bi-linear terms. The decoupling of the left and right modes is of course general to all half string bases. Splitting now to left and right part is obvious. The first term is K^l_1 and the second is K^r_1.

We can now use eq. (2.21) to calculate $\mathcal{L}_0 + \mathcal{L}_0^\dagger$ in the half-string basis, and then transform back to the κ basis to get the same result (1.10) we got in the previous subsection. In this calculation we get an infinite constant from the normal ordering of the operators. We study this constant later.

2.3 Non-zero momentum and the bosonized ghost sector

The operators $\mathcal{L}_0, \mathcal{L}_0^\dagger$ contain also a term linear in the momentum that we still did not calculate. This term would not contribute in the case of uniform tachyon condensation, because $p = 0$ in this case. However, it may be of importance in studying generalizations of Schnabl's solution in the context of lump solutions. Moreover, it is also important for describing the bosonized ghost sector.

The momentum dependent term in the matter sector is,[1]

$$\delta_1 L^{\rm m}_n = \sqrt{|n|} a_n p_0 \quad (n \neq 0) , \qquad \delta L^{\rm m}_0 = \frac{1}{2} p_0^2 , \tag{2.27}$$

which upon using (2.1) implies,

$$\delta_1 \mathcal{L}^{\rm m}_0 = \frac{1}{2} p_0^2 - 2p_0 \int_{-\infty}^\infty d\kappa \Big(\sum_{n=1}^\infty \frac{(-1)^n \sqrt{2n}\, v^\kappa_{2n}}{4n^2-1} \Big) \frac{a_\kappa}{\sqrt{\mathcal{N}(\kappa)}} . \tag{2.28}$$

Complex conjugation gives $\delta_1 \mathcal{L}^{\dagger\,\rm m}_0$. In the bosonized ghost sector, p_0 should be replaced by the half-integer ghost number q_0, giving $\delta_1 \mathcal{L}^{\rm g}_0$. In this case there are also additional contributions from the linear-dilaton character of the bosonized ghost,

$$\delta_2 L^{\rm g}_n = \frac{Q}{2}(n+1)\sqrt{|n|} a_n \quad (n \neq 0) , \qquad \delta_2 L^{\rm g}_0 = \frac{Q}{2} q_0 , \tag{2.29}$$

[1] We work in the $2\alpha' = 1$ conventions.

where $Q = -3$. Thus, in addition to $\delta_1\mathcal{L}_0^{\mathrm{g}}$, $\delta_1\mathcal{L}_0^{\dagger\,\mathrm{g}}$, which are complex conjugate to each other, we have in the ghost sector also,

$$\delta_2\mathcal{L}_0^{\mathrm{g}} = -\frac{3}{2}q_0 + 3\int_{-\infty}^{\infty} d\kappa \Big(\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{2n}\, v_{2n}^{\kappa}}{2n-1}\Big)\frac{a_\kappa}{\sqrt{\mathcal{N}(\kappa)}}\,, \tag{2.30}$$

$$\delta_2\mathcal{L}_0^{\dagger\,\mathrm{g}} = -\frac{3}{2}q_0 - 3\int_{-\infty}^{\infty} d\kappa \Big(\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{2n}\, v_{2n}^{\kappa}}{2n+1}\Big)\frac{a_\kappa^\dagger}{\sqrt{\mathcal{N}(\kappa)}}\,. \tag{2.31}$$

These are not complex conjugate to each other. We see that we have to evaluate the expressions inside the parentheses in these two equations. Their difference would then give (2.28). The problem is, of course, that these expressions do not converge.

In order to deal with this problem, we separate the sums to

$$\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{2n}\, v_{2n}^{\kappa}}{2n\pm 1} = \sum_{n=1}^{\infty}(-1)^n \frac{v_{2n}^{\kappa}}{\sqrt{2n}} \mp \sum_{n=1}^{\infty} \frac{(-1)^n}{2n\pm 1}\frac{v_{2n}^{\kappa}}{\sqrt{2n}}\,. \tag{2.32}$$

It was argued in [29] that the first term in the r.h.s, which is the divergent one, converges as a distribution to

$$\sum_{n=1}^{\infty}(-1)^n \frac{v_{2n}^{\kappa}}{\sqrt{2n}} = \mathcal{P}\frac{1}{\kappa}\,. \tag{2.33}$$

The evaluation of the two converging sums is straightforward, either by using directly the generating function, as was done in [29], or by substituting the integral representation of v_{2n}^{κ} [20],

$$v_{2n}^{\kappa} = \frac{(-1)^n\sqrt{2n}}{2\pi}\mathcal{N}(\kappa)\int_{-\infty}^{\infty} du \frac{\sin(\kappa u)\tanh^{2n-1}(u)}{\cosh^2(u)}\,. \tag{2.34}$$

Including (2.33) we get,

$$\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{2n}\, v_{2n}^{\kappa}}{2n-1} = \frac{\pi}{2}\mathcal{P}\coth\left(\frac{\kappa\pi}{2}\right), \tag{2.35}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{2n}\, v_{2n}^{\kappa}}{2n+1} = \frac{\pi}{2}\mathcal{P}\frac{1}{\sinh\left(\frac{\kappa\pi}{2}\right)}\,. \tag{2.36}$$

Thus the final result is,

$$\delta_1\mathcal{L}_0^{\mathrm{g}} = \delta_1\mathcal{L}_0^{\mathrm{m}} = \frac{1}{2}p_0^2 - p_0\frac{\pi}{2}\int_{-\infty}^{\infty} d\kappa \tanh\left(\frac{\kappa\pi}{4}\right)\frac{a_\kappa}{\sqrt{\mathcal{N}(\kappa)}}\,, \tag{2.37}$$

and the additional ghost terms are,

$$\delta_2\mathcal{L}_0^{\mathrm{g}} = -\frac{3}{2}q_0 + \frac{3\pi}{2}\int_{-\infty}^{\infty} d\kappa \mathcal{P}\coth\left(\frac{\kappa\pi}{2}\right)\frac{a_\kappa}{\sqrt{\mathcal{N}(\kappa)}}\,, \tag{2.38}$$

$$\delta_2\mathcal{L}_0^{\dagger\,\mathrm{g}} = -\frac{3}{2}q_0 - \frac{3\pi}{2}\int_{-\infty}^{\infty} d\kappa \mathcal{P}\frac{1}{\sinh\left(\frac{\kappa\pi}{2}\right)}\frac{a_\kappa^\dagger}{\sqrt{\mathcal{N}(\kappa)}}\,. \tag{2.39}$$

JHEP10(2006)067

3. Verifying the commutation relation

In this section, we evaluate the commutation relation (1.12). This can be considered as a test to our choice of the principal part prescription in section 2. We start with the zero-momentum case in 3.1, then we evaluate the linear terms and the bosonized ghost in 3.2.

3.1 The quadratic terms

The commutation relation, can be written more explicitly as,

$$[\mathcal{L}_{0,\text{bi-lin}},\mathcal{L}^\dagger_{0,\text{bi-lin}}]+[\mathcal{L}_{0,\text{quad}},\mathcal{L}^\dagger_{0,\text{quad}}]=\mathcal{L}_{0,\text{bi-lin}}+\mathcal{L}^\dagger_{0,\text{bi-lin}}\,, \tag{3.1}$$

$$[\mathcal{L}_{0,\text{bi-lin}},\mathcal{L}^\dagger_{0,\text{quad}}]=\mathcal{L}^\dagger_{0,\text{quad}}\,, \tag{3.2}$$

and the conjugate of the last equation. We start with,

$$[\mathcal{L}_{0,\text{quad}},\mathcal{L}^\dagger_{0,\text{quad}}]=\int_0^\infty d\kappa d\kappa' \frac{\frac{\kappa\pi}{2}\frac{\kappa'\pi}{2}}{\sinh\left(\frac{\kappa\pi}{2}\right)\sinh\left(\frac{\kappa'\pi}{2}\right)}(a_\kappa a^\dagger_{\kappa'}+a^\dagger_{-\kappa'}a_{-\kappa})\delta(\kappa-\kappa')\,. \tag{3.3}$$

We see that normal ordering this expression brings an infinite factor of the form of an integral over $\delta(0)$. This is not much of a surprise, since we are not working here with the full, $c=0$ theory, but rather with a $c=1$ one-dimensional matter sector. We deal with this infinite constant later. Ignoring the constant term, we write

$$[\mathcal{L}_{0,\text{quad}},\mathcal{L}^\dagger_{0,\text{quad}}]=\int_{-\infty}^\infty d\kappa \frac{\left(\frac{\kappa\pi}{2}\right)^2}{\sinh^2\left(\frac{\kappa\pi}{2}\right)}a^\dagger_\kappa a_\kappa\,. \tag{3.4}$$

Next we calculate,

$$\begin{aligned}
[\mathcal{L}_{0,\text{bi-lin}},\mathcal{L}^\dagger_{0,\text{bi-lin}}]=&\int_{-\infty}^\infty d\kappa d\kappa' d\kappa_1 d\kappa_1'\Big(\frac{\kappa\pi}{4}\coth\left(\frac{\kappa\pi}{2}\right)\delta(\kappa-\kappa')+\frac{\kappa+\kappa'}{2}\delta'(\kappa-\kappa')\Big)\cdot\\
\cdot\Big(\frac{\kappa_1\pi}{4}\coth\left(\frac{\kappa_1\pi}{2}\right)&\delta(\kappa_1-\kappa_1')+\frac{\kappa_1+\kappa_1'}{2}\delta'(\kappa_1-\kappa_1')\Big)\big(a^\dagger_\kappa a_{\kappa_1}\delta(\kappa'-\kappa_1')-a^\dagger_{\kappa_1'}a_{\kappa'}\delta(\kappa-\kappa_1)\big)\\
=\int_{-\infty}^\infty d\kappa \frac{\kappa\pi}{4}&\frac{\sinh(\kappa\pi)-\kappa\pi}{\sinh^2\left(\frac{\kappa\pi}{2}\right)}a^\dagger_\kappa a_\kappa\,.
\end{aligned} \tag{3.5}$$

In order to get to the result, we had to exchange the names of the indices $\kappa\leftrightarrow\kappa_1'$, $\kappa'\leftrightarrow\kappa_1$ for the expression multiplying the last delta function, evaluate the integral over κ_1' and use the identity,

$$\delta^{(n)}(x)x^k=\begin{cases}\frac{n!(-1)^k}{(n-k)!}\delta^{(n-k)}(x) & k\leq n\\ 0 & k>n\end{cases}\,. \tag{3.6}$$

It is immediate that (2.20) is indeed the sum of (3.4) and (3.5).

Finally, we have to verify (3.2),

$$[\mathcal{L}_{0,\text{bi-lin}},\mathcal{L}^\dagger_{0,\text{quad}}]=\pi\int_{-\infty}^\infty d\kappa d\kappa' \frac{\frac{\kappa\pi}{4}\coth\left(\frac{\kappa\pi}{2}\right)\delta(\kappa-\kappa')+\frac{\kappa+\kappa'}{2}\delta'(\kappa-\kappa')}{\mathcal{N}(\kappa')}a^\dagger_\kappa a^\dagger_{-\kappa'}\,. \tag{3.7}$$

Now, we use the invariance of the creation operators under $\kappa \leftrightarrow -\kappa'$, to write the second summand in a symmetric form,

$$\begin{aligned}
&\frac{\pi}{2}\int_{-\infty}^{\infty} d\kappa d\kappa' \frac{\kappa+\kappa'}{\mathcal{N}(\kappa')}\delta'(\kappa-\kappa')a^\dagger_\kappa a^\dagger_{-\kappa'} = \\
&\frac{\pi}{4}\int_{-\infty}^{\infty} d\kappa d\kappa'(\kappa+\kappa')\delta'(\kappa-\kappa')\Big(\frac{1}{\mathcal{N}(\kappa')}-\frac{1}{\mathcal{N}(\kappa)}\Big)a^\dagger_\kappa a^\dagger_{-\kappa'} \,.
\end{aligned} \tag{3.8}$$

Expanding this expression in $\kappa - \kappa'$ and using once again the identity (3.6), brings (3.7) exactly into (2.17).

3.2 The linear terms

The additional, momentum-dependent terms should obey some relations in order that the algebra (1.12) would still hold. In the matter sector[2] the relations are,

$$[\delta_1\mathcal{L}_0, \mathcal{L}_0^\dagger] = \delta_1\mathcal{L}_0 - \frac{1}{2}p_0^2 \,, \tag{3.9}$$

$$[\delta_1\mathcal{L}_0, \delta_1\mathcal{L}_0^\dagger] = p_0^2 \,. \tag{3.10}$$

Since all the expressions involved are regular, it is clear that the relations would hold and indeed they do, as can be seen from a straightforward calculation. In the ghost sector the coefficient functions are singular around $\kappa = 0$ and the integrals are not well defined. This should have also been expected, since the ghost contribution should cancel the infinite constant from the matter sector.

In the ghost sector some new relations emerge. Most of them are trivial like the relations that involve only δ_1. These are,

$$[\delta_1\mathcal{L}_0, \delta_2\mathcal{L}_0^\dagger] + [\delta_2\mathcal{L}_0, \delta_1\mathcal{L}_0^\dagger] = -3q_0 \,, \tag{3.11}$$

$$[\delta_2\mathcal{L}_0, \mathcal{L}_{0,\text{quad}}^\dagger] + [\mathcal{L}_{0,\text{bi-lin}}, \delta_2\mathcal{L}_0^\dagger] = \delta_2\mathcal{L}_0^\dagger + \frac{3}{2}q_0 \,, \tag{3.12}$$

$$[\delta_2\mathcal{L}_0, \mathcal{L}_{0,\text{bi-lin}}^\dagger] + [\mathcal{L}_{0,\text{quad}}, \delta_2\mathcal{L}_0^\dagger] = \delta_2\mathcal{L}_0 + \frac{3}{2}q_0 \,. \tag{3.13}$$

The only non-trivial relation is the one related to the normalization, which we examine next,

$$[\delta_2\mathcal{L}_0, \delta_2\mathcal{L}_0^\dagger] = -\Big(\frac{3\pi}{2}\Big)^2\int_{-\infty}^{\infty} d\kappa\,\mathcal{P}\frac{\coth\big(\frac{\kappa\pi}{2}\big)}{\sinh\big(\frac{\kappa\pi}{2}\big)\mathcal{N}(\kappa)} \,. \tag{3.14}$$

This expression diverges, since it has a double pole in the origin. However, in [29] a prescription was given for dealing with this type of divergences in the continuous basis. According to this prescription, we should interpret the following integral in level truncation as,

$$\int_{-\infty}^{\infty} d\kappa\,\mathcal{P}\frac{2}{\kappa^2\mathcal{N}(\kappa)} \approx \sum_{n=1}^{\ell/2}\frac{1}{n} \,, \tag{3.15}$$

[2]These relations are also part of the ghost sector relations, where we should write q_0 instead of p_0. We omit the superscripts m,g in the following expressions.

where ℓ is the oscillator level. Subtracting this expression inside the integral leaves us with a converging integral,

$$-\left(\frac{3\pi}{2}\right)^2\int_{-\infty}^{\infty}d\kappa\left(\frac{\coth\left(\frac{\kappa\pi}{2}\right)}{\sinh\left(\frac{\kappa\pi}{2}\right)}-\frac{2}{\pi^2}\frac{2}{\kappa^2}\right)\frac{1}{\mathcal{N}(\kappa)}=-\frac{9}{2}\left(2\log(2)-1\right). \tag{3.16}$$

Thus we write,

$$[\delta_2\mathcal{L}_0,\delta_2\mathcal{L}_0^\dagger]\approx-\frac{9}{2}\left(\sum_{n=1}^{\ell/2}\frac{1}{n}+2\log(2)-1\right). \tag{3.17}$$

We now want to add the contribution to the normalization coming from (3.3) of all 27 modes. Again, we have to regularize this expression, using the spectral density (2.5),

$$27[\mathcal{L}_{0,\text{quad}},\mathcal{L}_{0,\text{quad}}^\dagger]\approx 27\int_0^\infty d\kappa\rho(\kappa)\frac{\left(\frac{\kappa\pi}{2}\right)^2}{\sinh^2\left(\frac{\kappa\pi}{2}\right)}=\frac{9}{2}\left(\sum_{n=1}^{\ell/2}\frac{1}{n}+2\log(2)-\frac{1}{6}\right). \tag{3.18}$$

We can now add all the contributions to the central term in the commutator of the full matter+ghost operators. Recall that the terms proportional to the momentum (and ghost number) already canceled. Thus,

$$[\mathcal{L}_{0,\text{full}},\mathcal{L}_{0,\text{full}}^\dagger]=\mathcal{L}_{0,\text{full}}+\mathcal{L}_{0,\text{full}}^\dagger+\frac{15}{4}. \tag{3.19}$$

There is a discrepancy between this result and (1.12) which comes from using different regularization schemes. Schnabl did the calculation in the universal basis with $c=0$ Virasoro operators, while here we use the continuous basis regularization, which relies on oscillator level truncation. There are two potential problems here. The first is the use of $c\neq 0$ Virasoro algebra due to the separation to matter and ghost parts and the second is the use of oscillator level-truncation calculations, which can lead to anomalous results [25]. In fact, already the matter-ghost factorization of $\mathcal{L}_0,\mathcal{L}_0^\dagger$ may lead to problems, as was noticed by Schnabl already in [15]. There, these operators (called there $B,B^\dagger$) were used to produce the "unbalanced wedge states" and it was demonstrated that the inclusion of these states in the algebra may lead to normalization inconsistencies.

We evaluated this constant also in the discrete basis. The contribution of the linear part (3.17) is reproduced analytically. This also trivially matches the contribution that we get in a Virasoro-based level truncation, since oscillators at level ℓ emerge only from $\delta_2 L_\ell$ (2.29). The contribution of the quadratic part (3.18) was evaluated numerically. We found a finite part, which is half the one of (3.18) to a very high precision. Thus, the direct use of oscillator level-truncation almost produces the results of the continuous basis. We believe that the source of the discrepancy here is that the integrand of (3.3) contains a delta-function, which should also be interpreted as $\rho(\kappa)$. This effectively gives a product of two such factors. However, we do not know how to treat such a product. What is really needed is a better regularization $\rho(\kappa,\kappa')$ (2.5) that would allow such manipulations. A better regularization would presumably result in a vanishing constant. We currently study these issues.

JHEP10(2006)067

4. Wedge states

In this section we demonstrate that our expressions for $\mathcal{L}_0, \mathcal{L}_0^\dagger$ generate the wedge states via both relation (1.5) and (1.6). This is another verification of the principal part prescription used in deriving these expressions. Since we are working with a $c = 1$ system, there are infinite normalization factors, which we ignore.

4.1 Relating the $\mathcal{L}_0^\dagger$ and squeezed state representations

The operator $\mathcal{L}_0^\dagger$ contains terms of the form $a^\dagger a^\dagger$ and $a^\dagger a$. We can use techniques similar to those of the appendix of [30] to write,

$$e^{\frac{1}{2}a^\dagger \tilde{S} a^\dagger + a^\dagger V a} = e^{\frac{1}{2}a^\dagger S a^\dagger} e^{a^\dagger V a}\,, \tag{4.1}$$

where $\tilde{S}, S, V$ are matrices and get the relation,

$$S = \frac{e^{2V}-1}{2V}\tilde{S}\,. \tag{4.2}$$

This expression is defined as a power series in V. Thus, it is well defined even for a non-invertible V and since the Taylor expansion has a linear term, it has a unique inverse.

We want to rewrite the equality between (1.5) and (1.7) as a first order differential equation in n. Since both expression trivially match for the initial condition $n = 2$, this equation is equivalent to the original equality, when we use the known n dependence in both expressions (1.5), (1.7). To this end we derive both equations with respect to n,

$$-n\int_0^\infty d\kappa \partial_n T_n(\kappa) a_\kappa^\dagger a_{-\kappa}^\dagger |n\rangle = \int_{-\infty}^\infty d\kappa d\kappa' \big(a_\kappa^\dagger a_{\kappa'}^\dagger \tilde{S}(\kappa,\kappa') + a_\kappa^\dagger a_{\kappa'} V(\kappa,\kappa')\big)|n\rangle\,. \tag{4.3}$$

We can now plug (1.7) into the r.h.s of this equation to get an expression with only creation operators,

$$-n\int_0^\infty d\kappa \partial_n T_n(\kappa) a_\kappa^\dagger a_{-\kappa}^\dagger |n\rangle = \int_{-\infty}^\infty d\kappa d\kappa' \big(a_\kappa^\dagger a_{\kappa'}^\dagger \tilde{S}(\kappa,\kappa') + a_\kappa^\dagger a_{-\kappa'}^\dagger V(\kappa,\kappa') T_n(\kappa)\big)|n\rangle\,. \tag{4.4}$$

Now, the coefficients of all sets of creation operators should vanish separately.

Instead of just plugging our solution into (4.4), we consider an ansatz of the form of the solution that we found (1.11) and show that these equations are the only solution for the ansatz. Namely we consider,

$$\tilde{S}(\kappa,\kappa') = \tilde{S}(\kappa)\delta(\kappa-\kappa')\,, \tag{4.5}$$
$$V(\kappa,\kappa') = V_1(\kappa)\delta(\kappa+\kappa') + V_2(\kappa+\kappa')\delta'(\kappa+\kappa')\,. \tag{4.6}$$

This can be considered as an independent derivation for the expression of $\mathcal{L}_0$.

Integrating the δ' term gives,

$$\int_{-\infty}^\infty d\kappa d\kappa' V_2(\kappa+\kappa')\delta'(\kappa+\kappa') T_n(\kappa) a_\kappa^\dagger a_{\kappa'}^\dagger =$$

$$
\begin{aligned}
-&\int_0^\infty d\kappa\Big(2a^\dagger_\kappa a^\dagger_{-\kappa}\big(V_2(2\kappa)\partial_\kappa T_n(\kappa)+T_n(\kappa)V_2'(2\kappa)\big)+T_n(\kappa)V_2(2\kappa)\partial_\kappa(a^\dagger_\kappa a^\dagger_{-\kappa})\Big)= \\
-&\int_0^\infty d\kappa a^\dagger_\kappa a^\dagger_{-\kappa}V_2(2\kappa)\partial_\kappa T_n(\kappa)\,,
\end{aligned}
\tag{4.7}
$$

where in the last equality we had to integrate by part and thus, in order to ensure that the boundary terms vanish, we had to assume that $V_2(\kappa)$ times the eigenvalues of $a^\dagger_\kappa a^\dagger_{-\kappa}$ is not too singular as $\kappa\to 0,\infty$ and that $2\leq n$. We can now write (4.4) as an equation that should hold for all $\kappa>0$,

$$
-n\partial_n T_n(\kappa)=\tilde S(\kappa)+2V_1(\kappa)T_n(\kappa)-V_2(2\kappa)\partial_\kappa T_n(\kappa)\,. \tag{4.8}
$$

All that is left to do now is to plug in the explicit expression for $T_n(\kappa)$ (1.8) and expand (4.8) in a series with respect to n. Equating the first three non-trivial coefficients, which are functions of κ gives a unique choice for $\tilde S(\kappa), V_{1,2}(\kappa)$ that exactly matches (2.17), (2.19). Plugging these expressions into the full equation (4.8), proves that it is indeed a solution.

4.2 Relating the $\mathcal{L}_0+\mathcal{L}_0^\dagger$ and squeezed state representations

The algebra (1.12) was used in [1] in order to express the wedge states using the sum $\mathcal{L}_0+\mathcal{L}_0^\dagger$ (1.6). This expression seems very similar to (1.5). However, since the sum $\mathcal{L}_0+\mathcal{L}_0^\dagger$ is defined using matrices which are block diagonal in the κ basis, it is much easier to directly derive (1.7) from (1.6), since here there is no problem to use the methods of [30] for the evaluation.

We write $\mathcal{L}_0+\mathcal{L}_0^\dagger$ in a block diagonal form as,

$$
\mathcal{L}_0+\mathcal{L}_0^\dagger=A\left(a^\dagger\begin{pmatrix}0&1\\1&0\end{pmatrix}a^\dagger+a\begin{pmatrix}0&1\\1&0\end{pmatrix}a\right)+C\,a^\dagger\begin{pmatrix}1&0\\0&1\end{pmatrix}a \tag{4.9}
$$

where

$$
A=\frac{\kappa\pi}{4\sinh\left(\frac{\kappa\pi}{2}\right)}\,,\qquad C=\frac{\kappa\pi}{2}\coth\left(\frac{\kappa\pi}{2}\right)\,. \tag{4.10}
$$

It is clear that we can express,

$$
\begin{aligned}
&\exp\Big(t(\mathcal{L}_0+\mathcal{L}_0^\dagger)\Big)= \\
&\exp\big(\eta(t)\big)\exp\left(\alpha(t)\,a^\dagger\begin{pmatrix}0&1\\1&0\end{pmatrix}a^\dagger\right)\exp\left(\gamma(t)\,a^\dagger\begin{pmatrix}1&0\\0&1\end{pmatrix}a\right)\exp\left(\alpha(t)\,a\begin{pmatrix}0&1\\1&0\end{pmatrix}a\right).
\end{aligned}
\tag{4.11}
$$

Since all the matrices involved are commutative, we can use the appendix of [30] to immediately write differential equations for the unknown functions $\alpha(t),\gamma(t),\eta(t)$,

$$
\begin{aligned}
\dot\alpha&=A+2C\alpha+4A\alpha^2\\
\dot\gamma&=C+4\alpha A\\
\dot\alpha&=e^{2\gamma}A\\
\dot\eta&=2\mathrm{Tr}(A\alpha)\,,
\end{aligned}
\tag{4.12}
$$

with the initial conditions,

$$\alpha(0) = \gamma(0) = \eta(0) = 0\,. \tag{4.13}$$

It may seem that we have too many equations, but given the initial conditions, the first two imply the third. We also do not calculate the normalization η, since we check here only the matter sector. Substituting the explicit expressions (4.10) into the equation, the solution is immediate,

$$\alpha(t) = \frac{1}{2}\frac{\sinh\left(\frac{\kappa\pi}{2}t\right)}{\sinh\left(\frac{\kappa\pi}{2}(1-t)\right)}, \qquad \gamma(t) = \log\left(\frac{\sinh\left(\frac{\kappa\pi}{2}\right)}{\sinh\left(\frac{\kappa\pi}{2}(1-t)\right)}\right). \tag{4.14}$$

Substituting $t = -\frac{n-2}{2}$ we get

$$\alpha\left(-\frac{n-2}{2}\right) = \frac{T_n}{2}\,, \tag{4.15}$$

where T_n is defined in (1.8). Thus, (1.7) is reproduced, as we wanted to prove.

5. Conclusions

We demonstrated in this paper that the operators $\mathcal{L}_0, \mathcal{L}_0^\dagger$ have a simple form for scalar fields in the continuous basis. The expressions we found could be used in the study of background dependent applications related to Schnabl's solution, but not only. Schnabl's choice of coordinates and gauge can also simplify calculations around the perturbative vacuum, where again our expressions could be put to use.

It was natural to generalize our results to the bosonized ghost sector. It is usually the case that the bosonized ghosts are easier to handle in string field theory. Still, it would be interesting to generalize our results and write $\mathcal{L}_0^{\mathrm{g}}, \mathcal{L}_0^{\dagger\,\mathrm{g}}$ using the b, c ghosts, especially considering that Schnabl's solution explicitly relies on them.

Our construction suffers from regularization subtleties. It seems to us that the source of the problems is that the oscillator level truncation that is used for the regularization of the continuous basis is inconsistent. Therefore, some calculations, such as the evaluation of wedge states normalizations cannot be trusted. Thus, we find the construction of a consistent regularization scheme for the continuous basis highly desirable.

Acknowledgments

We would like to thank Dmitri Belov, Yaron Oz, Robertus Potting, Martin Schnabl and Stefan Theisen for discussions. The work of M. K. is supported by a Minerva fellowship. The work of E. F. is supported by the German-Israeli foundation for scientific research.

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JHEP10(2006)067